Evaluation of Numerical Models … HEC-RAS and DHI-MIKE 11

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ABSTRACT: In "Evaluation of Numerical Models of Flood and Tide Propagation" (ASCE Journal of Hydraulics Oct 2001), Rodney J. Sobey described six benchmark tests for unsteady flow model validation. In this paper, two programs that have been used extensively for flood stage prediction around the world, the Hydrologic Engineering Center's - River Analysis System (HEC- RAS) and the Danish Hydraulic Institute's (DHI) - MIKE 11 were applied to these benchmark tests. Both models performed well on the benchmark tests. In addition to the theoretical benchmarks, this paper also demonstrates that both models are capable of simulating observed transients in the California Aqueduct.

INTRODUCTION:

In "Evaluation of Numerical Models of Flood and Tide Propagation" (Journal of Hydraulic Engineering/October 2001), Sobey suggests that an "extensive and independent review … should be a routine and automatic part of any numerical model study." Sobey outlines six hydrodynamic tests for one-dimensional, free surface numerical models, and provides sample output with his own ESTFLOW program. The tests were designed to demonstrate the capabilities of the applied code and to expose any deficiencies of the formulation or solution of the basic mass and momentum equations, equations (1) and (2) respectively.

$$b\frac{\partial \eta}{\partial t} + \frac{\partial Q}{\partial x} - q = 0 \qquad (1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) = -gA\frac{\partial \eta}{\partial x} - \frac{\tau_o}{\rho}p \qquad (2)$$

In this paper the ESTFLOW solutions are compared with the two models most commonly used in flood stage prediction around the world, the public domain software of the Hydrologic Engineering Center's - River Analysis System (**HEC-RAS**) and the proprietary software of the Danish Hydraulic

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Institute's (DHI) - **MIKE 11**. Surprisingly, neither model has received peer-reviewed publication of its efficacy in the past. In addition to the theoretical benchmarks, this paper also contains an application to the San Joaquin Aqueduct in California where the transients were measured after an abrupt gate closing.

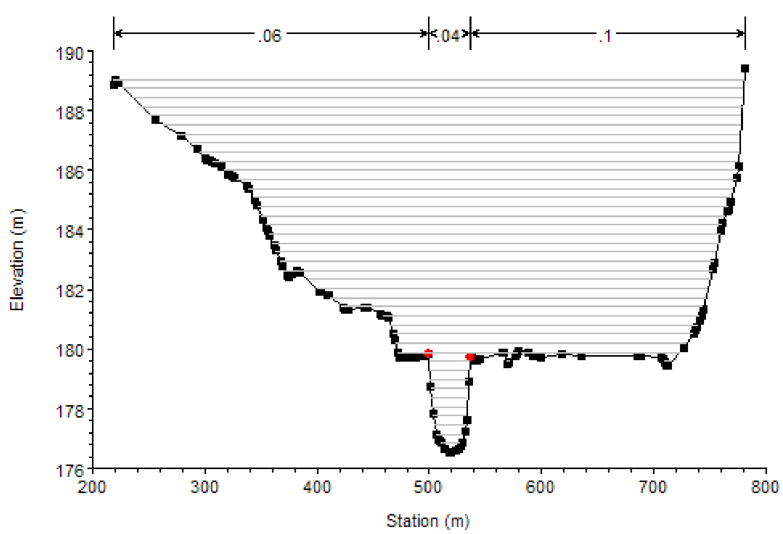
The models are introduced with a short discussion of the computational methodologies. In several of the benchmark tests, Sobey presented the solutions as individual terms of the continuity and momentum equations. The respective developers modified both HEC-RAS and MIKE 11 to output the terms of the differential equations in order to provide a comparison with the benchmarks.

The ESTFLOW code used by Sobey to demonstrate his benchmark tests uses the method of characteristics to solve the mass and momentum equation describing gradually varied, unsteady flow. The method of characteristics uses a technique to change the underlying partial differential equations into a set of ordinary differential equations that are solved using common Runge-Kutta solution techniques. The exact formulation is detailed in Sobey's paper.

General computational information common to HEC-RAS and MIKE 11

HEC-RAS and MIKE 11 are both general one dimensional (cross-section integrated) unsteady, open channel, hydraulics programs. Both programs have modern graphical user interfaces with extensive plots and tables to assist in setting up models and viewing output. They are capable of modeling natural cross sections at irregular spacing (though all of Sobey's benchmark tests use trapezoidal cross sections at regular spacing). They have interfaces for commonly used GIS packages, which can be used to extract geometric data from a terrain surface and delineate a floodplain from the computed water surfaces.

In preparation for unsteady computations, the programs compute tables of hydraulic properties, such as flow area and top width as a function of water surface elevation. The tables are used to speed the unsteady computations, see the sample cross section and table shown in Figure 1.



Elevation (m)	Area (m2)	Top Width (m)
176.85	3.76	17.61
177.15	10.55	25.44
177.46	18.76	28.22
177.76	27.72	30.34
178.07	37.15	31.40
178.37	46.85	32.28
178.68	56.83	33.16
178.98	67.06	34.01
179.29	77.58	35.00
179.59	89.16	46.02
⋮	⋮	⋮
188.74	3750.96	553.27
189.04	3921.06	563.19
189.34	4092.80	563.75

Figure 1. Sample cross section divided vertically for table of properties

Internal boundaries, such as bridges and culverts, are preprocessed into a family of rating curves that describe the head loss through a structure for a given tail water and flow. The hydraulics of internal boundaries that change during the simulation, such as gated structures, are computed during the simulation.

HEC-RAS specific computational methodology information

The computation engine for the HEC-RAS program is based on the U.S. Army Corp of Engineer's (USACE) model Unsteady Network Model (UNET, Barkau, 1992). The program solves the mass conservation and momentum conservation equations with an implicit linearized system of equations using Preissman's second order box scheme. In a cross section, the overbank and channel are assumed to have the same water surface, though the overbank volume and conveyance are separate from the channel volume and conveyance in the implementation of the conservation of mass and momentum equations. The simultaneous system of equations generated for each time step (and iterations within a time step) are stored with a skyline matrix scheme and reduced with a direct solver developed specifically for unsteady river hydraulics by Dr. Robert Barkau.

The state variables for the numerical scheme are flow and stage, which are computed and stored at each cross section. Plots of flow and stage are available for selected cross sections at a user specified time interval.

The hydraulic resistance is based on the friction slope from the empirical Manning's equation, with several ways of modifying the roughness. Roughness can be characterized with Manning's (n) or roughness height's (k).

DHI-MIKE 11 specific computational methodology information

The MIKE 11 solution of the continuity and momentum equations is based on an implicit finite difference scheme developed by Abbott and Ionescu (1967). The scheme is setup to solve any form of the Saint Venant equations – i.e. kinematic, diffusive, or dynamic. The water level and flow are calculated at each time step, by solving the continuity equation and the momentum equation using a 6-point Abbot scheme with the mass equation centered on h-points and the momentum equation centered on Q-points. By default, the equations are solved with 2 iterations. The first iteration starts from the results of the previous time step and the second uses the centered values from the first iteration. The number of iterations is user specified.

Cross sections are easily specified in both area and longitudinal location through the user interface. The water level (h points) is calculated at each cross section and at model interpolated interior points located evenly and specified by the user-entered maximum distance. The flow (Q) is then calculated at points midway between neighboring h-points and at structures.

The hydraulic resistance is based on the friction slope from the empirical equation, Manning's or Chezy, with several ways of modifying the roughness to account for variations throughout the cross-sectional area.

BENCHMARK TESTS

The benchmark tests are divided into three categories, modeling a single channel (C1-C3), modeling a network (N1-N2), and modeling an inline hydraulic structure (S1). Each test has been designed so that the stable solution has a Courant number less than one, providing conditions within the capabilities of a practical numerical code. Please refer to Sobey's original paper for a more detailed description of the individual test cases.

Case C1: Steady, Uniform Flow in Open-Ended Channel

The first benchmark examines how computed water surfaces transition from elevated initial conditions to normal depth. A 3000 m trapezoidal channel starts with an initial water surface above normal depth, and at the first time step, the downstream stage is lowered to normal depth. ESTFLOW, HEC-RAS, and MIKE 11 model the transition to normal depth. The C1 benchmark test is designed to expose any math formulation, whether any issues resolving the non-linear terms exist, or if there is simply a coding error in the program.

The initial and boundary conditions are listed in Table 1 and the layout is diagramed in Figure 2. The solutions from the hydraulic models are compared at a cross-section 900 m from the downstream end of the channel and the individual terms of the continuity equation and momentum equation are plotted over the simulation (Figures 3 and 4). The units of the continuity equation are in m^2/s and the units of the momentum equation are in m^3/s^2.

Table 1. C1 benchmark specifications

x_F	Δx	x_L	z_F	z_L	BW	SS	t_F	Δt	Δt_{output}	t_L
0	150 m	3,000 m	+1 m	+2 m	3 m	2	0	30 s	30 s	30 min

Initial Conditions: $\eta(x, 0) = z(x) + 4.0$ m and $Q(x, 0) = 60\ m^3/s$;
Boundary Conditions: $\eta(x_F, t) = z(x_F) + h_n$ and $Q(x_L, t) = 50\ m^3/s$;
n = 0.02.

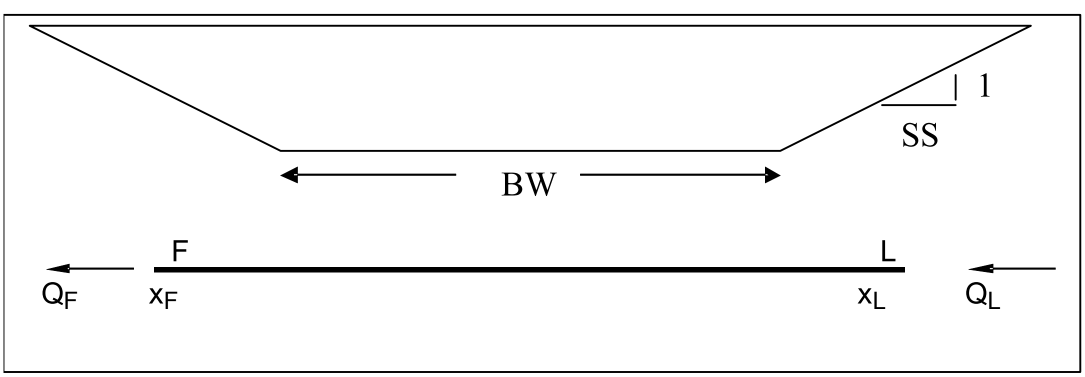


Figure 2. C1 benchmark channel and reach sketch

In descriptions of all benchmarks x_F designates the longitudinal position of first cross section (downstream), Δx the space step, x_L the longitudinal position of last cross section (upstream), z_F the downstream bed elevation, z_L the downstream bed elevation, BW the bottom width, SS the side slopes (horizontal to 1 vertical), t_F the time at beginning of simulation, Δt the computational time step, Δt_{output} the model output time step, t_L the time at end of simulation, $\eta(x, 0)$ the initial stage equation, $z(x)$ the bed elevation at x, $Q(x, 0)$ the initial flow rate, $\eta(x_F, t)$ the downstream stage boundary condition, h_n the normal depth, $Q(x_L, t)$ the upstream flow boundary condition and n the Manning's friction coefficient.

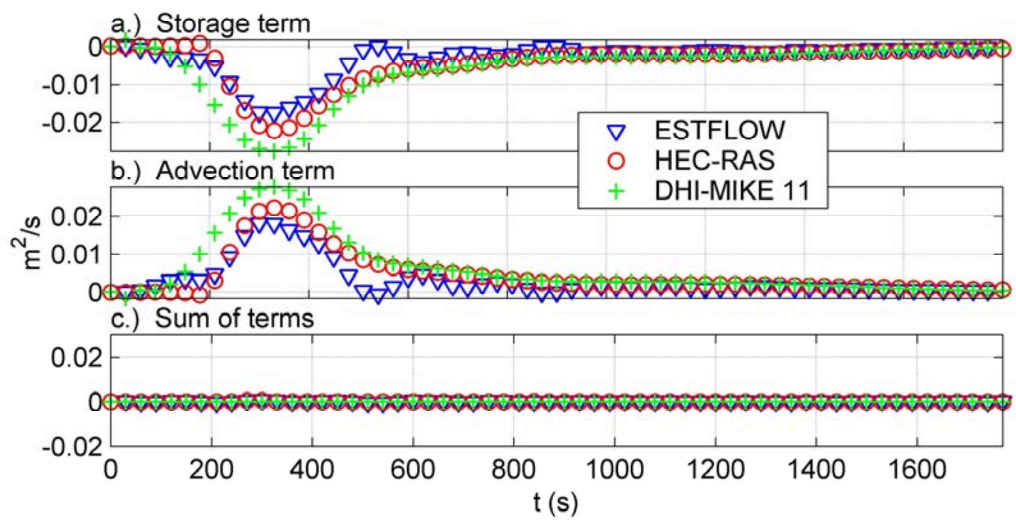


Figure 3. C1 benchmark continuity equation terms

As noted in Sobey's analysis, the 150 m cross-section spacing and a 30-second time step provide a rough discretization of the problem. All models pass this difficult transient and go to normal depth while preserving mass and momentum. If one desired to track this difficult drop in flow and stage, a smaller time and space step would be required for these types of numerical schemes. Nothing unusual is demonstrated in the graphs of the individual equation terms of the mass and momentum equations and all models show reasonable conservation.

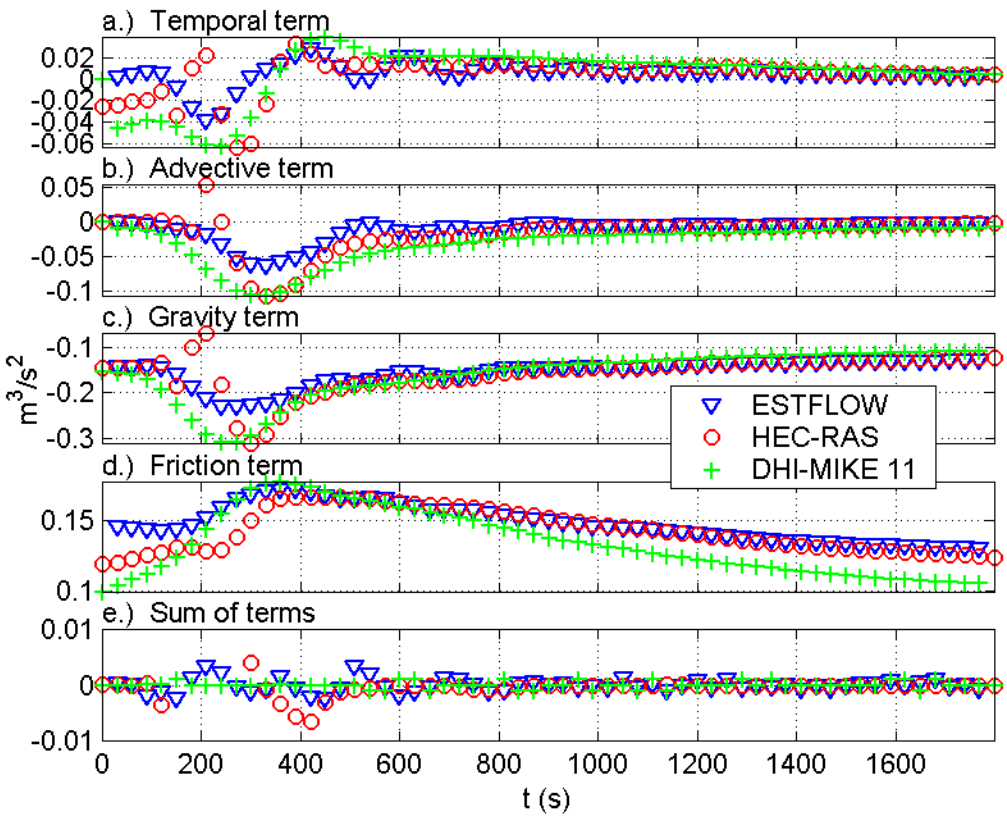


Figure 4. C1 benchmark momentum equation terms

Case C2: Transient Evolution of Initial Mound

The second benchmark is a single reach that has a mound shaped profile for initial conditions that spreads upstream and downstream during the unsteady simulation. The initial and boundary conditions for this test are in Table 2. The numerical solutions, and a simplified analytical solution for this benchmark, were compared with profiles at various times in the simulation (Figure 5). The simplified analytical solution models the movement of a wave, but does not account for friction losses or boundary conditions.

Table 2. C2 benchmark specifications

x_F	Δx	x_L	z_F	z_L	BW	SS	t_F	Δt	Δ_{output}	t_L
0	150 m	4,500 m	+0 m	+0 m	3 m	2	0	30 s	30 s	25 min

Initial Conditions: $\eta(x, 0) = 2 + 0.2 \exp\{-c[(x - 2{,}250)/500]^2\}$ and $Q(x, 0) = 0$ m^3/s;
Boundary Conditions: $Q(x_F, t) = 0$ m^3/s and $\eta(x_L, t) = +2$ m; $n = 0.02$: $c = \log 2$.

The mound from the initial conditions disperses in both directions and the numerical models follow the simplified analytical solution for 360 seconds and then start to separate after the boundary conditions start to impact the solution field (the analytical solution is not constrained by these boundary conditions). The downstream boundary stage can change but the upstream stage is fixed. Both HEC-RAS and MIKE 11 reproduce the ESTFLOW solution, the differences after 1500 seconds are probably due to the calculation of frictional loss methodologies used in the various numerical schemes.

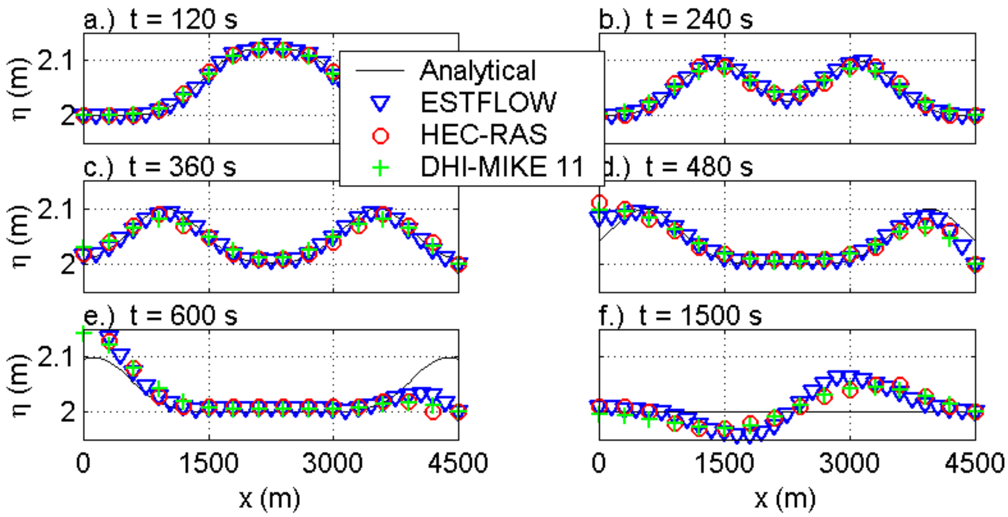


Figure 5. Output comparisons for benchmark C2 with stage profiles for each model at 6 times in the simulation. The initial conditions and solution remains symmetrical until influenced by boundary conditions

Case C3: Transition to Steady-State Tidal Circulation

The C3 benchmark uses a single reach similar to the previous tests, but has a time varying downstream stage and is designed to find problems with the propagation of a tidal forcing function as well as the initial start-up transients. A long prismatic channel with a constant (zero) flow is subjected to a periodic stage boundary condition on the downstream end. The initial and boundary conditions are provided in Table 3. The computed stage and flow at a cross section 3000 m from the moving boundary are shown in Figure 6.

Table 3. C3 benchmark specifications

x_F	Δx	x_L	z_F	z_L	BW	SS	t_F	Δt	Δt_{output}	t_L
0	250 m	10,000 m	-5 m	-5 m	50 m	0	0	30 s	10 min	25 hr

Initial Conditions: $\eta(x, 0) = 0$ m and $Q(x, 0) = 0$ m^3/s;
Boundary Conditions: $\eta(x_F, t) = 0.5 \sin \omega t$ m and $Q(x_L, t) = 0$ m^3/s;
$n = 0.02$; $\omega = 2\pi/12.5\ hr^{-1} = 2\pi/(12.5 \times 3.600)\ s^{-1}$.

The signature of the tidal stage boundary affects the entire reach, and the flows have transients for the first tidal cycle. All models have nearly identical results with this test case predicting the transition from still water to a regular tidal cycle. The hydrodynamics are dominated by the inertial terms in the momentum balance; all three models utilize the same formulation for the inertial term and accordingly provide very similar answers.

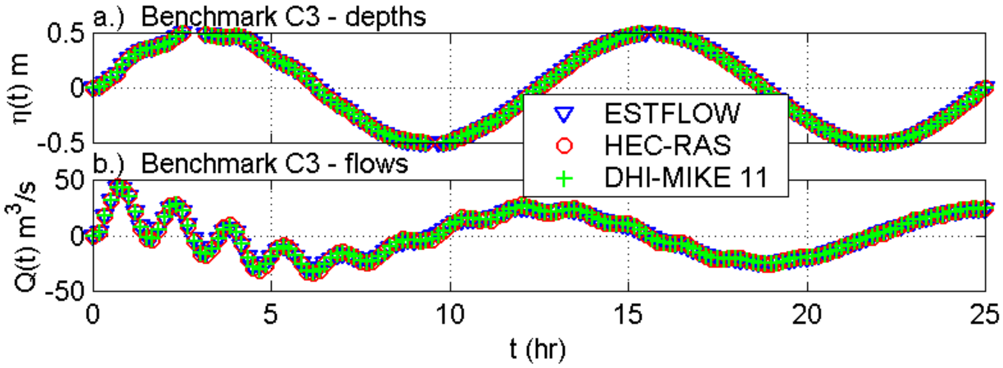


Figure 6. Output comparison for benchmark C3 at x = 3000 m

Case N1: Steady Flow through Channel Network

The N1 benchmark is the first network test case, and is diagramed in Figure 7. The test is analogous to C1, in that the simulation is an examination of the transition to steady state from perturbed initial conditions. The initial and boundary conditions are described in Table 4. The model solutions of the computed stages at the internal junctions are compared in Figure 8 and the flow through the node "B" confluence is compared in Figure 8.

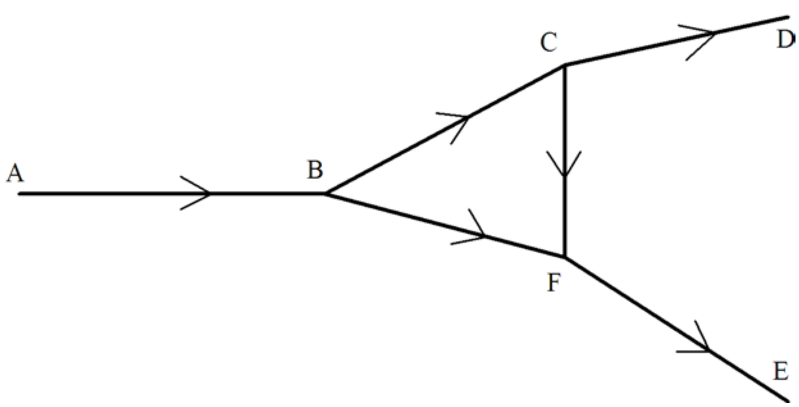


Figure 7. Network Schematic for benchmarks N1 and N2

Table 4. N1 benchmark specifications

Δx	t_F	Δt	Δt_{output}	t_L
150 m	0	15 s	180 s	6 hr

Initial Conditions: $\eta(x, 0) = +0.5$ m and $Q(x, 0) = 0$ m^3/s;
Boundary Conditions: $\eta_A(t) = 0$ m and $Q_D(t) = 100$ m^3/s and $Q_E(t) = 50$ m^3/s;
Q_D and Q_E introduced gradually by linear ramp function over 15 min.

Where η_A = stage at node A, Q_D= flow into node D, and Q_E = flow into node E.

The magnitude of the scale for the flow balance in Figure 9a has been exaggerated to show differences; the differences are minor and all preserve flow well at node B.

The data presented in Figure 8 and Figure 9 demonstrate all three models are in close agreement. However, in Figure 8a and in Figure 9b, c, & d the MIKE 11 results show a significantly greater initial response near node B. The MIKE 11 code is providing an improved resolution of the event, for which both ESTFLOW and HEC-RAS would require shorter time steps and cross-sectional distances to accomplish.

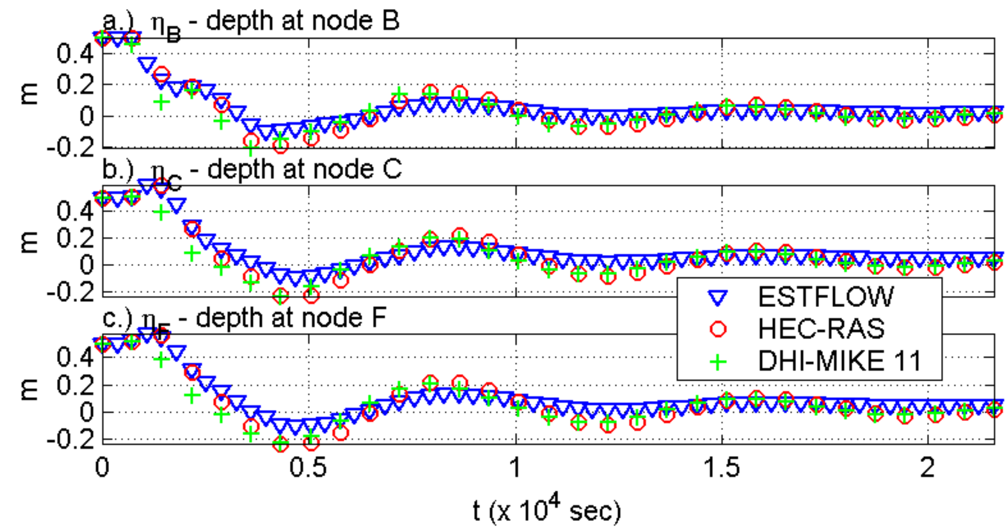


Figure 8. Output comparison of stage for benchmark N1 at nodes B, C, and F

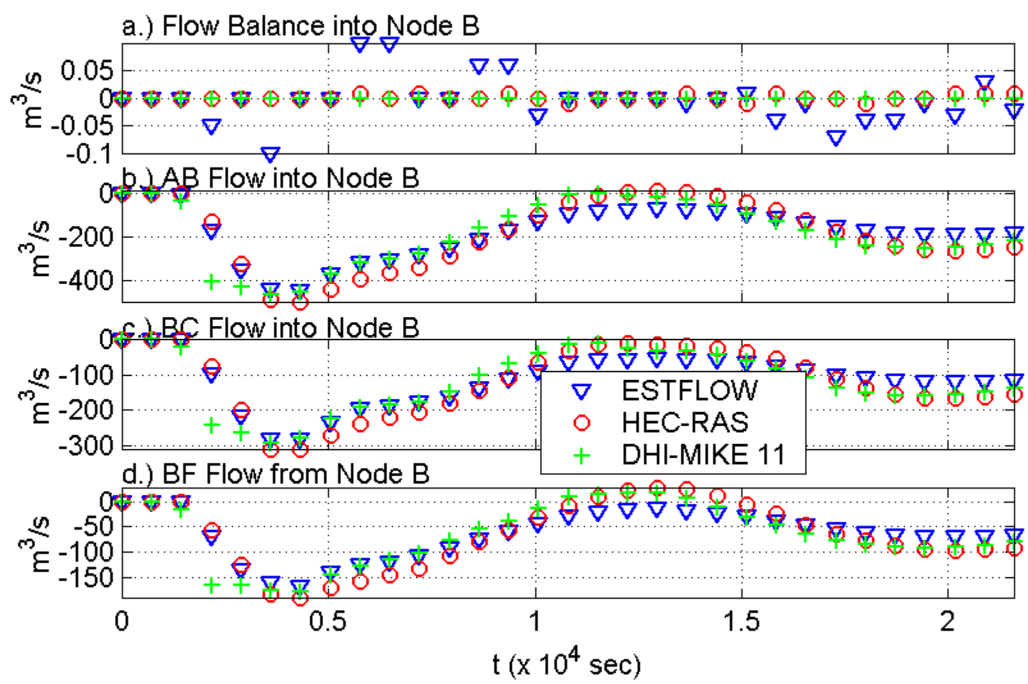


Figure 9. Output comparison of flow for benchmark N1 at node B

Case N2: Unsteady Flow through Channel Network

The second network benchmark uses the geometry from the first, but adds a periodic stage boundary at the downstream end of the system. While under the steady-state conditions of case N1, the inertia term of the momentum equation is zero and balanced only by gravity and friction. The addition of the tidal boundary condition introduces inertia complications back into the solution.

The initial and boundary conditions are listed in Table 5. The stage and flow solutions from the three models are compared in Figure 10.

Table 5. N2 benchmark specifications

Δx	t_F	Δt	Δt_{output}	t_L
150 m	0	15 s	600 s	2 T

Initial Conditions: Steady-state solution from N1;
Boundary Conditions: $\eta_A(t) = a_A \sin \omega t$ m and $Q_D(t) = 100$ m^3/s and $Q_E(t) = 50$ m^3/s;
$a_A = 0.5$ m, $\omega = 2\pi/T$, and tidal period T = 12.5 hr.

The computed stages at each end of the cross over channel (CF) in the network are all nearly identical. The computed flows for HEC-RAS and MIKE 11 are also nearly identical, but do not match the ESTFLOW solution due only to differences in computations of friction.

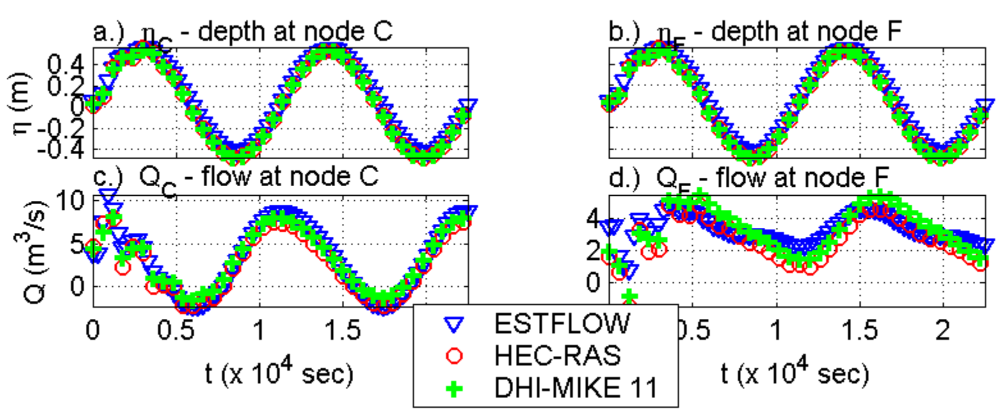


Figure 10. Output comparisons for benchmark N2. Stage and flow from the cross sections at the ends of reach CF

Case S1: Transition to Steady Flow through Gate

The S1 benchmark test explores modeling an internal boundary condition. The initial transients in a long trapezoidal reach with an inline weir are examined. The initial and boundary conditions are in Table 6. The computed profiles of this test are asymptotically moving toward a steady state solution. The model's predictions are compared in Figure 11 and Figure 12 with profiles after 270 and 1170 seconds of simulation.

Table 6. S1 benchmark specifications

x_F	Δx	x_L	z_F	z_L	BW	SS	t_F	Δt	Δt_{output}	t_L
0	150 m	15,000 m	-2 m	-2 m	3 m	2	0	30 s	90 s	1.5 hr

Initial Conditions: $\eta(x, 0) = 0$ m and $Q(x, 0) = 0$ m^3/s;
Boundary Conditions: $\eta(x_L, t) = 0$ m and $Q(x_F, t) = +100$ m^3/s;
Q_F introduced by a linear ramp function over 30 s;
Gate, $x_B = x_C = 6000$ m, W = 2.5 m, $z_{sill} = -1.5$ m, $C_G = 0.5$; n = 0.02

This test case has initial transients from two sources, from the rapid change in flow from 0 → 100 m^3/sec and from the internal forced relationship between stage and flow over the gate. HEC-RAS, MIKE 11 and ESTFLOW produce a relatively similar propagation of the flow transition. The ESTFLOW solution, as noted in the original paper, has regular oscillations and is explained as the expected free mode responses bouncing off the internal boundary (gate). HEC-RAS and MIKE 11 would require a smaller time and distance step to characterize this type of transient. The case study presented next demonstrates that HEC-RAS and MIKE 11 are capable of modeling internal waves caused by a gate closure in a real system.

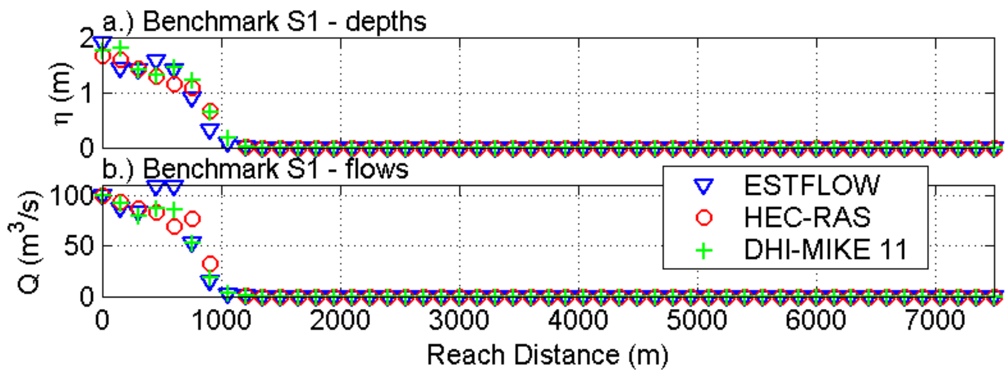


Figure 11. Profile comparison for benchmark S1 after 270 seconds

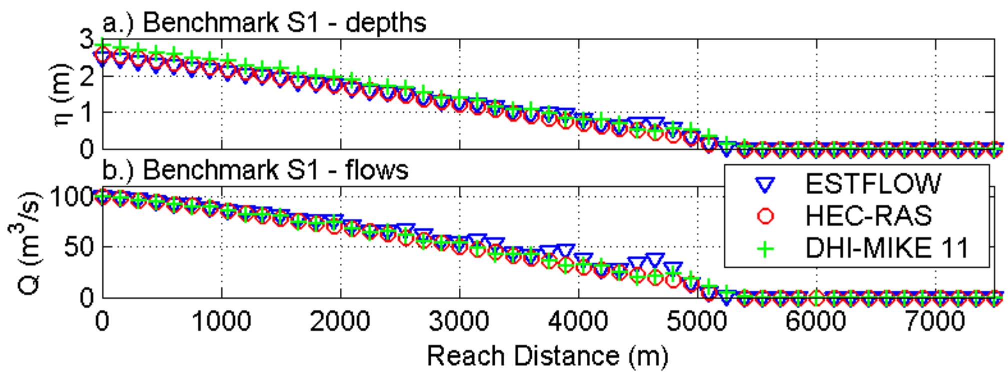


Figure 12. Profile comparison for benchmark S1 after 1170 seconds

Case Study – San Joaquin Canal

The San Joaquin Canal is part of the agricultural distribution system in the Central Valley Project (CVP) of California. The canal is a cascade of channels (referred to as pools by CVP) separated by gated structures; the stage is measured at the upstream and downstream ends of the pool. The goal of the models is to predict an observed transient generated from sequentially closing the gates on both ends of one of the pools in the canal (DeVries et al., 1968).

Table 7. San Joaquin Canal specifications

x_F	Δx	x_L	z_F	z_L	BW	SS	t_F	Δt	Δt_{output}	t_L
0	15.2 m	8,944 m	60.40 m	60.82 m	12.2 m	1.5	0	10 s	60 s	120 min

Gate size: Height = 6.1 m, Width = 12.2 m;
Initial Conditions: $\eta(x, 0) = 68.82$ and $Q(x, 0) = 48.1$ m^3/s;
Boundary Conditions: $\eta(x_F, t) = 70.0$ m and
$Q(x_L, t) = 48.14$ m^3/s for $t < 10$ min
$Q(x_L, t) = 0$ m^3/s for $t > 11$ minutes with linear drop over 1 min;
Gate (at x_F) Open Height = 0.80 m for $t < 22$ min and closes over 4 min;
n = 0.016.

Wave Celerity and Expected Travel Time in the Canal:
Average Hydraulic Depth = 5.33 m
Wave Celerity = 7.23 m/s
Travel Time ~ 20.5 min
Round Trip Time ~ 41 min

The stage along the entire pool reach is shown in Figure 13 for 6 times. The upstream gate closure (t = 10 min) causes a drop in the stage at the upstream end of the pool that travels downstream. Before the wave reaches the end of the pool, the downstream gate starts to close (t = 22 min), which also creates a run up wave. Both waves combine on the downstream end of the canal and then reflect back to the upstream gate. The wave reflects off the upstream gate and travels with a round trip time of 41 minutes.

The computed water surface elevations are compared to the observed water surface elevations at the gages at the upstream and downstream end of the pool in Figure 14. Both models do a good job in predicting the stage changes at each end of the pool reach.

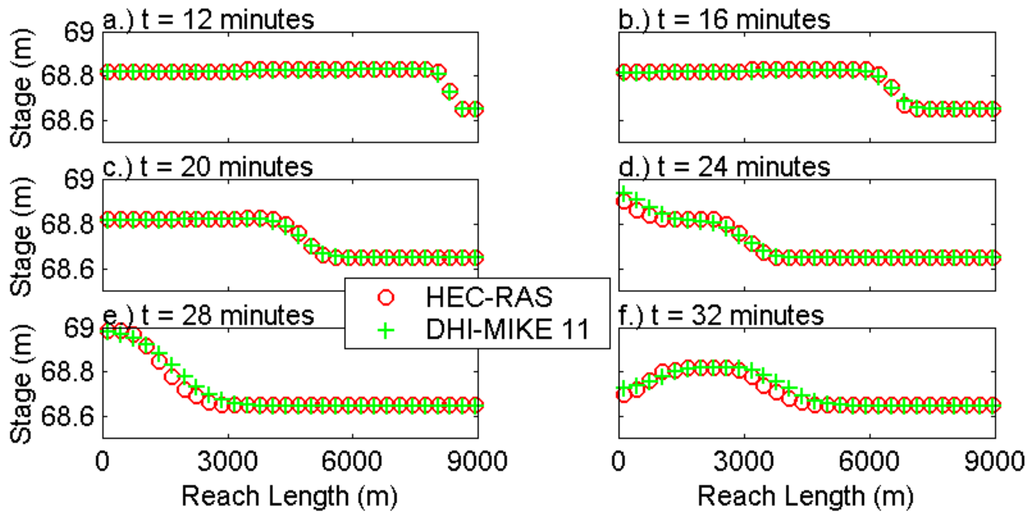


Figure 13. Stage along the entire reach at 6 times

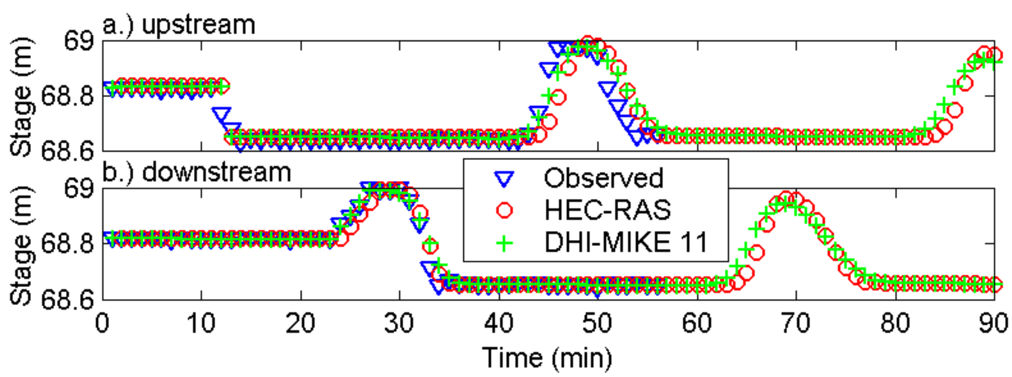


Figure 14. Model predictions compared to observed values at the upstream and downstream ends of the reach

CONCLUSIONS:

Sobey developed a series of benchmark tests for unsteady, one-dimensional, open channel flow models. The tests ranged from simple single reach models to looped networks. Boundary conditions were designed to be at the edge of expected stability. In spite of computational differences, both the HEC-RAS and MIKE 11 models successfully demonstrated the ability to model these benchmark cases. For the rough discretization in some of the benchmark tests, the MIKE 11 code demonstrated an ability to respond more quickly to disturbances presented by the initial conditions.

In addition to the theoretical tests, HEC-RAS and MIKE 11 were applied to a measured transient generated from closing gates in the San Joaquin canal. Both models performed well in simulating the observed magnitude and speed of the transient waves as they combined and were reflected back and forth along the canal.

Properly applied to one-dimensional modeling projects, both HEC-RAS and MIKE 11 will produce viable predictions.

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NOTATION:

The following symbols are sued in this paper:

A = local cross-sectional area of channel;
BW = bottom width;
b = local surface width of channel;
g = gravitational acceleration;
n = Manning friction coefficient;
Q = discharge;
S = slope;
SS = side slopes (horizontal to 1 vertical);
s = seconds;
T = period;
t = time;
Δt = time step;
x = longitudinal position;
Δx = space step;
z = bed elevation;
η = water surface elevation;
τ = shear stress; and
ω = radian frequency.

Subscripts

F = first;
L = last;
N = mode, timestep;
n = normal depth; and
0 = bed.